Exercises for Error and distance

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1. Binary code words from the code $\{000, 010, 101, 110, 111\}$ are sent over a binary symmetric channel (BSC) with cross-over probability $p = 0.02$. Decode using the maximum likelihood decoding rule the following words: 001, 011 and 001.

2. Let a memoryless binary channel have channel probabilities $p_{00} = 0.8$ and $p_{11} = 0.9$, where p_{ij} is the probability that j is received when i is sent. Suppose the code words being sent over this channel are from the binary code $\{000, 100, 110, 011, 111\}$. Decode the words 001, 010 and 101 with the use of the maximum likelihood decoding rule.

3. Consider a binary code $C = \{010, 110, 101\}$.

a. Use the nearest neighbour decoding rule to decode the received word 000.

b. Let our channel be binary and memoryless with the probabilities $p_{00} = 0.2$ and $p_{11} = 0.5$. Decode the received word 000 using the maximum likelihood decoding rule.

4. Decode using the nearest neighbour decoding rule for the binary code

$$C = \{10110, 11000, 10100, 10011, 11011\}$$

the received words 00000, 00011, 01101, 01111 and 10011.

5. Use the nearest neighbour decoding rule to decode for the ternary code

$$C = \{01122, 10021, 20210, 22200\}$$

the received code words 00122, 12001, 20111 and 22000.

6. Construct the incomplete maximum likelihood decoding (IMLD) table for the binary code $C = \{000, 010, 101, 110, 111\}$.

7. Find the number of binary $(n, 2, n)$-codes, $n \geq 2$, where for a (n, m, d)-code n is the length of the word, m the size of the dictionary and d the distance of the code.

8. Consider the binary repetition code of length 6 sent over a binary symmetric channel which has symbol error probability p. Find the word error probability of the code.

9. Consider q-ary $(3, m, 2)$-codes, where $q \geq 2$. Find the range which m may take.

10. Let $A_q(n, d)$ represent the largest value of m such that there exists a q-ary (n, m, d)-code. Find $A_q(n, 1)$ and $A_q(n, n)$. Then find $A_q(3, 2)$ for any integer $q \geq 2$.

11. Find the upper bound of m for the q-ary $(q + 1, m, 3)$-code.

12. Consider a balanced block (b, v, r, k, λ)-design, where b is the number of subsets B_i of a set S of v elements, each point appears in exactly r blocks, each block comprises exactly k points, and each pair of points occurs together in exactly λ blocks. Here B_i are call *blocks*, and S is said to contain v *varieties*. Show that $bk = vr$ and $r(k - 1) = \lambda(v - 1)$

13. A *permutation* of a set S is a one-to-one mapping from S to itself. Two q-ary codes are said to be *equivalent* to each other if one can be obtained from the other by permutation of the positions of the code, or permutation of the symbols appearing in a fixed position, or any combination of both. Show that the binary codes $C_1 = \{00100, 00011, 11000, 11111\}$ and $C_2 = \{00000, 01101, 10110, 11011\}$ are equivalent. Then show that the ternary code $C_3 = \{012, 120, 201\}$ is equivalent to the ternary repetition code of length 3, $C_4 = \{000, 111, 222\}$.

14. Prove that a sphere of radius r in $\mathbf{F}_q^n$, $0 \leq r \leq n$, contains exactly

$$\binom{n}{0} + \binom{n}{1}(q-1) + \cdots + \binom{n}{r}(q-1)^r$$

vectors.

Reference

Raymond Hill. *A first course in coding theory.* Clarendon, 1986

San Ling and Chaoping Xing. *Coding theory, a first course.* Cambridge University Press, 2004